# Ab Initio Second-Order Møller-Plesset Calculation of the Vibrational Spectrum of Tetrahedrane 

B. Andes Hess, Jr.,* and L. J. Schaad*<br>Contribution from the Department of Chemistry, Vanderbilt University, Nashville, Tennessee 37235. Received July 9, 1984


#### Abstract

The SCF vibrational spectra, including IR intensities, have been computed for tetrahedrane and tetrahedrane- $d_{4}$ with a $6-31 \mathrm{G}^{*}$ basis. Møller-Plesset second-order correlation corrections to the frequencies also have been computed.


There has been much recent interest in the $\mathrm{C}_{4} \mathrm{H}_{4}$ potential surface as shown by the successful preparation of two long-sought isomers, cyclobutadiene ${ }^{1-3}$ and methylenecyclopropene. ${ }^{4,5}$ In the case of cyclobutadiene, theory ${ }^{6-12}$ has played an important role in confirming its synthesis and structure.

One of the most interesting $\mathrm{C}_{4} \mathrm{H}_{4}$ isomers, tetrahedrane (1), has so far eluded isolation although its tetra-tert-butyl derivative has been prepared and characterized by Maier. ${ }^{13}$ Extensive calculations by Kollmar ${ }^{14}$ have suggested that because of a high degree of strain as little as $10 \mathrm{kcal} / \mathrm{mol}$ may be required to sever a $\mathrm{C}-\mathrm{C}$ bond in tetrahedrane which may therefore be isolable only at quite low temperatures. If this turns out to be the case, lowtemperature IR and Raman spectroscopy are likely to be useful in the eventual characterization of this compound. We therefore report here an ab initio calculation of the vibrational frequencies and the IR intensities of tetrahedrane.


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A 4-31 G ab initio calculation of the vibrational frequencies together with an INDO estimate of IR intensities was reported as early as 1974 by Schulman and Venanzi. ${ }^{15}$ However, our experience ${ }^{11.12,16}$ indicates that inclusion of polarization functions and correlation energy greatly improves the agreement between theory and experiment. Calculations were therefore performed
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Table I. Optimized Geometry of Tetrahedrane ${ }^{a}$

| basis set | $R_{\mathrm{C}-\mathrm{C}}$ | $R_{\mathrm{C}-\mathrm{H}}$ | energy |
| :--- | :--- | :--- | :---: |
| $4-31 \mathrm{G}^{b}$ | 1.482 | 1.054 | -153.3414 |
| $\mathrm{DZ}+\mathrm{P}^{c}$ | 1.460 | 1.064 | -153.4537 |
| $6-31 \mathrm{G}^{*}$ | 1.463 | 1.063 | -153.5979 |
| $\mathrm{DZ}+\mathrm{P} /$ CEPA $^{c}$ | 1.470 |  | -154.0453 |
| $6-31 \mathrm{G}^{*} / \mathrm{MP}^{c}$ | 1.477 | 1.073 | -154.1009 |

${ }^{a}$ Bond lengths in $\AA$ and energies in au. ${ }^{b}$ Reference 15. ${ }^{c}$ Reference 14.

Table II. Symmetry Coordinates ${ }^{a, b}$

$$
\begin{aligned}
& \text { species } \quad \text { coordinate } \\
& S_{2}=1 / 2\left(R_{15}+R_{26}+R_{37}+R_{48}\right) \\
& \text { E } \quad S_{3}=\left[1 / 2\left(3^{1 / 2}\right)\right]\left(2 R_{12}-R_{23}-R_{13}-R_{14}-R_{24}+2 R_{34}\right) \\
& S_{4}=\left[1 / 2\left(6^{1 / 2}\right)\right]\left(2 A_{251}-A_{362}-A_{173}-A_{184}-A_{462}-A_{273}-\right. \\
& \left.A_{284}-A_{451}-A_{351}+2 A_{384}+2 A_{473}+2 A_{162}\right) \\
& S_{3^{\prime}}=1 / 2\left(R_{23}-R_{13}+R_{14}-R_{24}\right) \\
& S_{4^{\prime}}=\left[1 / 2\left(2^{1 / 2}\right)\right]\left(A_{362}-A_{173}+A_{184}-A_{462}+A_{273}-A_{284}+\right. \\
& \left.A_{451}-A_{351}\right) \\
& \mathrm{T}_{1} \quad S_{5}=\left[1 / 2\left(2^{1 / 2}\right)\right]\left(A_{251}+A_{173}+A_{462}-A_{284}-A_{351}+A_{384}-\right. \\
& \left.A_{473}-A_{162}\right) \\
& S_{5^{\prime}}=\left[1 / 2\left(2^{1 / 2}\right)\right]\left(-A_{251}-A_{362}-A_{184}+A_{273}+A_{451}+A_{384}-\right. \\
& \left.A_{473}+A_{162}\right) \\
& S_{5^{\prime \prime}}=\left[1 / 2\left(2^{1 / 2}\right)\right]\left(-A_{362}-A_{173}+A_{184}+A_{462}+A_{273}-A_{284}\right. \\
& \left.-A_{451}+A_{351}\right) \\
& \mathrm{T}_{2} \quad S_{6}=\left(1 / 2^{1 / 2}\right)\left(R_{23}-R_{14}\right) \\
& S_{7}=1 / 2\left(R_{15}+R_{48}-R_{37}-R_{26}\right) \\
& S_{8}=\left[1 / 2\left(2^{1 / 2}\right)\right]\left(A_{251}+A_{384}-A_{473}-A_{162}+A_{351}+A_{284}-\right. \\
& \left.A_{173}-A_{462}\right) \\
& S_{6^{\prime}}=\left(1 / 2^{1 / 2}\right)\left(-R_{13}+R_{24}\right) \\
& S_{7^{\prime}}=1 / 2\left(-R_{26}-R_{48}+R_{37}+R_{15}\right) \\
& S_{8^{\prime}}=\left[1 / 2\left(2^{1 / 2}\right)\right]\left(-A_{362}-A_{184}+A_{273}+A_{451}+A_{251}-A_{162}-\right. \\
& \left.A_{384}+A_{473}\right) \\
& S_{6^{\prime \prime}}=\left(1 / 2^{1 / 2}\right)\left(-R_{12}+R_{34}\right) \\
& S_{7^{\prime \prime}}=1 / 2\left(-R_{37}+R_{26}-R_{48}+R_{15}\right) \\
& S_{8^{\prime \prime}}=\left[1 / 2\left(2^{1 / 2}\right)\right]\left(-A_{173}+A_{462}-A_{284}+A_{351}+A_{451}-A_{273}\right. \\
& \left.+A_{362}-A_{184}\right)
\end{aligned}
$$

${ }^{a}$ The bond between atoms $i$ and $j$ is denoted by $R_{i j}$ and the angle formed by atoms $i, j$, and $k$ with $j$ at the vertex by $A_{i j k}$. ${ }^{b}$ Unprimed, primed, and doubly primed coordinates transform respectively like first, second, or third columns of the corresponding representation.
with $6-31 \mathrm{G}^{* 17}$ and $6-31 \mathrm{G}^{*} / \mathrm{MP}^{18}$ wave functions. Our ability to perform MP2 computations on a molecule of this size was greatly enhanced by the availability of an MP2 program written to take advantage of symmetry. ${ }^{19}$

Optimized geometries and energies are compared with several previous calculations in Table I. Kollmar ${ }^{8,14}$ has found tetrahedrane to lie $27 \mathrm{kcal} / \mathrm{mol}$ above cyclobutadiene on the $\mathrm{C}_{4} \mathrm{H}_{4}$ potential surface with a DZ + P basis set and $29 \mathrm{kcal} / \mathrm{mol}$ with inclusion of correlation. We find the same value at the SCF level

[^0]Table III. Calculated Force Constants ${ }^{a}$

| species | constant | SCF/6-31G* | MP2/6-31 G* |
| :---: | :---: | :---: | :---: |
| $\mathrm{A}_{1}$ | $\mathrm{~F}_{11}$ | 4.898 | 4.391 |
|  | $\mathrm{~F}_{22}$ | 6.778 | 6.387 |
|  | $\mathrm{~F}_{12}$ | -0.042 | -0.104 |
| E | $\mathrm{F}_{33}$ | 6.342 | 5.450 |
|  | $\mathrm{~F}_{44}$ | 0.276 | 0.218 |
|  | $\mathrm{~F}_{34}$ | 0.644 | 0.649 |
| $\mathrm{~T}_{1}$ | $\mathrm{~F}_{55}$ | 0.328 | 0.256 |
| $\mathrm{~T}_{2}$ | $\mathrm{~F}_{66}$ | 6.230 | 5.450 |
|  | $\mathrm{~F}_{77}$ | 6.774 | 6.387 |
|  | $\mathrm{~F}_{88}$ | 1.133 | 0.867 |
|  | $\mathrm{~F}_{67}$ | -0.153 | -0.277 |
|  | $\mathrm{~F}_{68}$ | 0.844 | 0.725 |
|  | $\mathrm{~F}_{78}$ | 0.023 | 0.035 |

${ }^{a}$ Stretching force constants are in mydn $\AA^{-1}$ and bending force contants in mdyn $\AA \mathrm{rad}^{-2}$.

Table IV. Vibrational Frequencies of Tetrahedrane ${ }^{a}$

|  | $\mathrm{C}_{4} \mathrm{H}_{4}$ |  |  | $\mathrm{C}_{4} \mathrm{D}_{4}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| species | $6-31 \mathrm{G}^{*}$ | $6-31 \mathrm{G}^{*} / \mathrm{MP} 2$ | $6-31 \mathrm{G}^{*}$ | $6-31 \mathrm{G}^{*} / \mathrm{MP} 2$ |  |
| $\mathrm{~A}_{1}$ | 1580 | 1494 | 1473 | 1393 |  |
|  | 3560 | 3460 | 2701 | 2625 |  |
| E | 701 | 556 | 526 | 425 |  |
|  | 934 | 878 | 880 | 813 |  |
| $\mathrm{~T}_{1}$ | 1015 | 888 | 814 | 713 |  |
| $\mathrm{~T}_{2}$ | $895(1.00)$ | 796 | $658(1.00)$ | 580 |  |
|  | $1291(0.05)$ | 1172 | $1232(0.26)$ | 1131 |  |
|  | $3513(0.10)$ | 3410 | $2600(0.25)$ | 2515 |  |

${ }^{a}$ Frequencies in $\mathrm{cm}^{-1}$; relative IR intensities given in parentheses.
(6-31G*), but with MP2 this energy difference decreases to 23 $\mathrm{kcal} / \mathrm{mol}$ (the disagreement may be due to the fact that Kollmar optimized only C-C bond distances in both molecules).
For the evaluation of force constants at the SCF level, the analytic gradient was computed for plus and minus distortions ( $0.01 \AA$ and $3^{\circ}$ ) along each symmetry coordinate (Table II). However, for the computation of $6.31 \mathrm{G}^{*} / \mathrm{MP} 2$ force constants both numerical first and second derivatives had to be calculated. Plus and minus distortions were made for the diagonal force constants and appropriate combination distortions for the offdiagonal force constants. The force constants that were obtained are given in Table III. Vibrational frequencies were computed with the Miyazawa version of Wilson's GF method. ${ }^{20}$ For the IR-active bands $\left(\mathrm{T}_{2}\right)$ derivatives of the dipole moment were obtained and used to compute IR intensities. Results are summarized in Table IV for tetrahedrane and its tetradeuterio derivative.

In Figure 1 are plotted tetrahedrane frequencies from the $4-31 \mathrm{G}, 6-31 \mathrm{G}^{*}$, and $6-31 \mathrm{G}^{*} / \mathrm{MP} 2$ computations. It is seen that there is a significant change in the computed pattern when polarization functions are added $\left(6-31 \mathrm{G}^{*}\right)$ with the order of bands being interchanged in two cases. As was found for the vibrational

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Figure 1. Theoretical vibrational frequencies of tetrahedrane.


Figure 2. Theoretical ( $6-31 \mathrm{G}^{*}$ ) IR spectra of tetrahedrane and tetradeuteriotetrahedrane. Relative intensities are shown by bar heights.
spectrum of cyclobutadiene, the main effect of correlation is to move all bands to lower frequency. Of the vibrations shown in Figure 1, only the $\mathrm{T}_{2}$ modes are IR active; and Figure 2 shows $6-31 G^{*}$ IR spectra (frequencies and intensities) for tetrahedrane and tetradeuteriotetrahedrane. In light of the excellent agreement found between the computed ${ }^{11}$ and experimental ${ }^{3}$ IR spectra of cyclobutadiene and methylenecyclopropene, ${ }^{4,21}$ we anticipate that the computed spectra reported here will be useful in the eventual identification of tetrahedrane.
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